PG – 819

II Semester M.Sc. Degree Examination, June 2015
(RNS) (2011-12 and Onwards)
MATHEMATICS
M – 204 : Partial Differential Equations

Time : 3 Hours   Max. Marks : 80

***Instruction** : Answer **any five full** questions. Choosing atleast **two** from **each** part.*

PART – A

1. a) Explain the method of characteristics for solving semilinear/quasilinear/non-linear first order PDEs. **6**

   b) Solve $xu_x + yu_y = xe^{-u}$ with the data $u = 0$ on $y = x^2$. **5**

   c) Obtain the solution of $(y - u)u_x + xyu_y + xu = 0$ using the method of characteristics. **5**

2. Using the method of characteristics or otherwise solve the following :

   a) $(u^2 - y^2)u_x + xyu_y + xu = 0$ with $u = y = x$, $x > 0$. **5**

   b) $u_t + uu_x = 0$, $t > 0$ with $u(x, 0) = \begin{cases} 1 & \text{for} \quad x \le 0 \\ 1 - 2x & \text{for} \quad 0 \le x \le 1 \\ -1 & \text{for} \quad x \ge 1 \end{cases}$. **5**

   c) $u_x u_y = u$ with $u(0, y) = y^2$. **6**

3. a) Classify

   i) $u_{xx} + xu_{yy} = 0$, $x \neq 0$ and

   ii) $(n - 1)^2 u_{xx} - y^{2n} u_{yy} = ny^{2n-1} u_y$ as being hyperbolic or parabolic or elliptic. Reduce the equations to their canonical forms. **(5+6)**

   b) Obtain the general solution of $u_{xxx} + u_{xxy} - u_{xyy} - u_{yyy} = e^{2x+y} + \sin(x + 2y)$ by any method. **5**

4. Explain Monge's method of solving $F(x, y, z, p, q, r, s, t) = 0$ and using the method obtain the solution of the equation $r + 4t = 4xy$. **16**

P.T.O.

PART – B

5. a) Arrive at the classical D' Alembert solution of the wave equation

$$\frac{\partial^2 u}{\partial t^2} = C^2 \frac{\partial^2 u}{\partial x^2}\ ;\ -\infty < x < \infty, t \geq 0$$

subject to
$$\left.\begin{aligned} u(x,0) &= f(x) \\ \frac{\partial u}{\partial t}(x,0) &= g(x) \end{aligned}\right\} -\infty < x < \infty$$

Use an appropriate Fourier transform for the purpose. 8

b) Using the method of separation of variables obtain the general solution of the three-dimensional wave equation in cylindrical polar coordinate system. 8

6. Solve the Dirichlet and Neumann problems involving the Laplace equation in the upper half-plane. 16

7. a) Illustrate Duhamel's principle using an IBVP involving a one-dimensional diffusion equation. 8

b) Using the method of separation of variables obtain the solution of the three-dimensional diffusion equation in spherical polar coordinates. 8

8. a) Obtain a three-term homotopy-perturbation general solution of he IBVP.

$$u_{tt} = \frac{x^2}{2} u_{xx}$$

$$u(x, 0) = x$$

$$\frac{\partial u}{\partial t}(x,0) = x^2$$ 6

b) Explain similarity transformation through an appropriate example. 6

c) Write down a weak formulation of

a) $u_{xx} + u_{yy} = x^2 + y^2$ and

b) $u_{xx} + u_{yy} + a^2 u = 0$ (a : constant). 4